AP Calculus AB
Test: Bridge to Calculus
Instructor's Version

Free-Response Question 1—no calculator (worth 15 points)

1. For each of the following, find:

a. $\lim_{x \to a^-} f(x)$

b. $\lim_{x \to a^+} f(x)$

c. $\lim_{x \to a} f(x)$

Show your work/justify your answers.

A. $f(x) = \begin{cases} 3x+1 & x < 2 \\ 8 & x = 2 \\ x^2+3 & x > 2 \end{cases}$ a=2

Answer:

a. $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (3x+1) = 7$ (1 point)

b. $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^2+3) = 7$ (1 point)

c. $\lim_{x \to 2} f(x) = 7$ because the one-sided limits are equal. (1 point)

(Maximum 2 points if no justification is given for answers.)

B. $f(x) = \dfrac{3-x}{|x-3|}$ a=3

Answer:

a. Since $f(x) = \begin{cases} -1 & x > 3 \\ 1 & x < 3 \end{cases}$, $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (1) = 1$. (1 point)

b. $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (-1) = -1$ (1 point)

c. $\lim_{x \to 3} f(x)$ does not exist because the one-sided limits are not equal. (2 points)

(Maximum 2 points if no justification is given for answers.)

© Copyright 2009 Apex Learning Inc. All rights reserved. This material is intended for the exclusive use of registered users only. No portion of these materials may be reproduced or redistributed in any form without the express written permission of Apex Learning Inc.

C. $f(x)=\dfrac{\sqrt{3+x}-\sqrt{3}}{x},\ a=0$

$$f(x)=\frac{\sqrt{3+x}-\sqrt{3}}{x},\frac{\sqrt{3+x}+\sqrt{3}}{\sqrt{3+x}+\sqrt{3}}=\frac{1}{\sqrt{3+x}+\sqrt{3}}$$

Answer:

a. $\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^-}\left(\dfrac{1}{\sqrt{3+x}+\sqrt{3}}\right)=\dfrac{1}{2\sqrt{3}}$ (1 point)

b. $\lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0^+}\left(\dfrac{1}{\sqrt{3+x}+\sqrt{3}}\right)=\dfrac{1}{2\sqrt{3}}$ (1 point)

c. $\lim\limits_{x\to 0} f(x)=\lim\limits_{x\to 0}\left(\dfrac{1}{\sqrt{3+x}+\sqrt{3}}\right)=\dfrac{1}{2\sqrt{3}}$ because the one-sided limits are equal. (2 points)

(Maximum 2 points if no justification is given for answers.)

D. $f(x)=\begin{cases}\sin\dfrac{x}{2}, & x<\dfrac{2\pi}{3}\\ \dfrac{x^2}{\pi}, & x\geq\dfrac{2\pi}{3}\end{cases}\qquad a=\dfrac{2\pi}{3}$

Answer:

a. $\lim\limits_{x\to\frac{2\pi^-}{3}} f(x)=\lim\limits_{x\to\frac{2\pi^-}{3}}\left(\sin\dfrac{x}{2}\right)=\sin\dfrac{\pi}{3}=\dfrac{\sqrt{3}}{2}$ (1 point)

b. $\lim\limits_{x\to\frac{2\pi^+}{3}} f(x)=\lim\limits_{x\to\frac{2\pi^+}{3}}\left(\dfrac{x^2}{\pi}\right)=\dfrac{4\pi}{9}$ (1 point)

c. $\lim\limits_{x\to\frac{2\pi}{3}} f(x)$ does not exist because the one-sided limits are not equal. (2 points)

(Maximum 2 points if no justification is given for answers.)

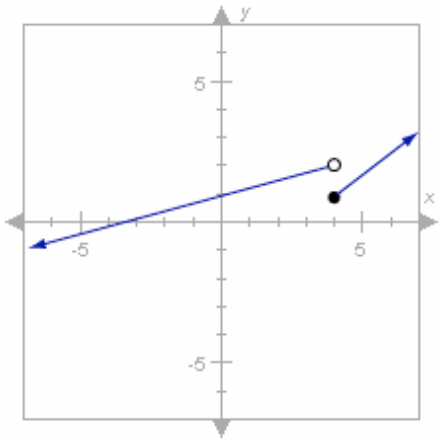
© Copyright 2009 Apex Learning Inc. All rights reserved. This material is intended for the exclusive use of registered users only. No portion of these materials may be reproduced or redistributed in any form without the express written permission of Apex Learning Inc.

Free-Response Question 2—with calculator (worth 15 points)

2. Sketch a graph of each function described.

A. A function that is continuous everywhere except at x=4. At x=4 the function is continuous from the right but not continuous from the left.

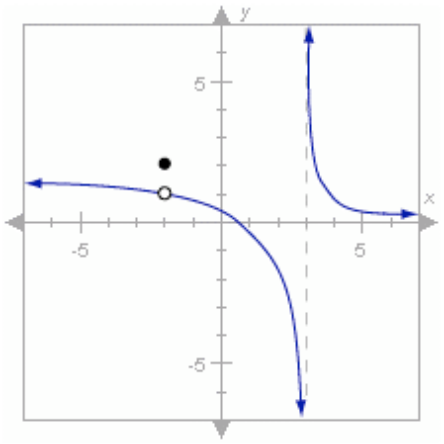
Answer:



(3 points maximum; 1 point for discontinuity at 4, and 2 points for continuity from the right at 4)

B. A function that has a removable discontinuity at x=-2 and an infinite discontinuity at x=3, but is continuous elsewhere.

Answer:

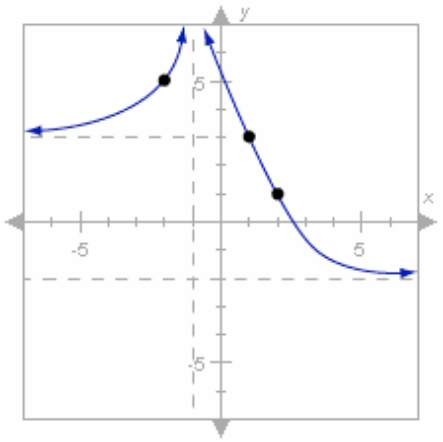


(3 points maximum; 1 point for vertical asymptote at 3, and 2 points for removable discontinuity at -2)

© Copyright 2009 Apex Learning Inc. All rights reserved. This material is intended for the exclusive use of registered users only. No portion of these materials may be reproduced or redistributed in any form without the express written permission of Apex Learning Inc.

C. A function such that:
$\lim_{x\to\infty} f(x) = -2$, $\lim_{x\to-\infty} f(x) = 3$, $f(1) = 3$, $f(2) = 1$, $f(-2) = 5$, $\lim_{x\to-1} f(x) = \infty$

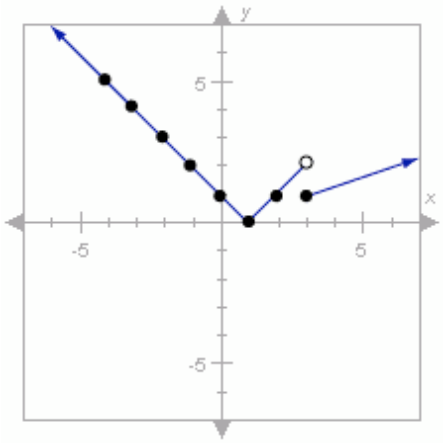
Answer:



(4 points maximum; 1 point for each horizontal asymptote [2 points total], 1 point for plotting all three points, and 1 point for the vertical asymptote. Award partial credit where appropriate.)

D. Graph $f(x) = \begin{cases} \sqrt{x-2} & x \geq 3 \\ |x-1| & x < 3 \end{cases}$ and determine if $f(x)$ is continuous at 3. Use limits to justify your answer.

Answer:



(1 point for the graph)

$f(x)$ is not continuous at 3 (1 point) because $\lim_{x\to3^-} f(x) \neq f(3)$ (2 points; other valid justifications may be awarded full credit) (4 points maximum points for Part D)

© Copyright 2009 Apex Learning Inc. All rights reserved. This material is intended for the exclusive use of registered users only. No portion of these materials may be reproduced or redistributed in any form without the express written permission of Apex Learning Inc.